

Potpourri

In-Class Problems

Math Circle Competition Team

August 27th, 2017

1. The average of five distinct positive integers is 85, and the average of the three largest of the integers is 100. Compute the largest possible value of the second-smallest integer.
2. At Pascal Prep, 10% of the students take calculus and 15% of the students wear flip-flops. After some study, it turns out that 40% of the students who wear flip-flops take calculus. If a given student takes calculus, compute the probability that they wear flip-flops.
3. Compute the sum of all real values of x such that $(4^{x-1})^{x-3} = 8^x$.

4. In rectangle $ABCD$, $AB = 20$ and $AD = 15$. Points M and N lie on \overline{BC} and \overline{CD} , respectively, and $[ABM] = [MCN] = [NDA]$ (where $[ABM]$ denotes the area of triangle ABM). Compute $[AMN]$.

5. Compute the least positive integer n such that $n!$ is divisible by 2015^2 .

6. If $1 + r + r^2 + \dots = 17$, compute $1 + 2r + 3r^2 + 4r^3 + \dots$.

7. (**Proof-based.** Please explain each step used in solving this problem.) In square $ABCD$, E is the midpoint of \overline{BC} , and F is the midpoint of \overline{CD} . Let G be the intersection of \overline{AE} and \overline{BF} . Prove that $DG = AB$.

