

Probability Problem Set

Math Circle Competition Team

September 24th, 2017

*1. Find n if $\binom{10}{4} + \binom{10}{3} = \binom{n}{4}$.

*2. (1988 AHSME) For any real number a and any positive integer k , define

$$\binom{a}{k} = \frac{a(a-1)(a-2)\cdots(a-(k-1))}{k!}.$$

What is the exact numerical value of $\frac{\binom{-1/2}{100}}{\binom{1/2}{100}}$?

*3. Find the number of zeroes at the end of $\binom{200}{100}$.

*4. (2004 HMMT Feb. Guts) Find the value of

$$\binom{6}{1}2^1 + \binom{6}{2}2^2 + \binom{6}{3}2^3 + \binom{6}{4}2^4 + \binom{6}{5}2^5 + \binom{6}{6}2^6.$$

*5. Find the coefficient of x^4y in the expansion of $(2x - 3y)^5$.

6. What is the coefficient of x^2 in the expansion of $\left(x + \frac{1}{x^2}\right)^{20}$?

7. Pat writes all the 7-digit numbers in which all the digits are different and each digit is greater than the one to its right (so the tens digit is greater than the units, the hundreds greater than the tens, and so on). For example, 9,865,320 is one of the numbers that Pat writes down.

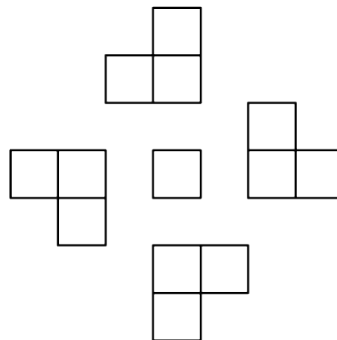
(a) How many numbers does Pat write down?

(b) One of Pat's numbers is chosen at random. What is the probability that the tens digit is a 1?

(c) One of Pat's numbers is chosen at random. What is the probability that the middle (thousands) digit is a 5?

*8. (2016 HMMT Nov. General) I have five different pairs of socks. Every day for five days, I pick two socks at random without replacement to wear for the day. Find the probability that I wear matching socks on both the third day and the fifth day.

- *9. **(2009 AMC 10A)** Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?
10. **(2010 AMC 10A)** Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?
11. **(2010 AIME II)** Dave arrives at an airport which has twelve gates arranged in a straight line with exactly 100 feet between adjacent gates. His departure gate is assigned at random. After waiting at that gate, Dave is told the departure gate has been changed to a different gate, again at random. Let the probability that Dave walks 400 feet or less to the new gate be a fraction $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
12. **(2013 AIME I)** In the array of 13 squares shown below, 8 squares are colored red, and the remaining 5 squares are colored blue. If one of all possible such colorings is chosen at random, the probability that the chosen colored array appears the same when rotated 90 degrees around the central square is $\frac{1}{n}$, where n is a positive integer. Find n .



13. A playoff series between two teams proceeds one game at a time until one team has won 5 games. What is the probability that the series lasts 9 games if each team is equally likely to win each game?
14. **(2016 HMMT Nov. General)** The numbers $1, 2, \dots, 11$ are arranged in a line from left to right in a random order. It is observed that the middle number is larger than exactly one number to its left. Find the probability that it is larger than exactly one number to its right.