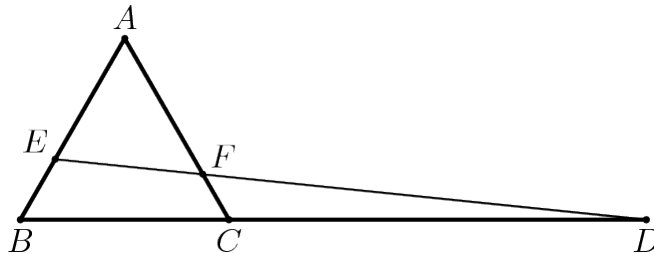


Geometry Day 2 Problem Set

Math Circle Competition Team

October 15th, 2017

1. In triangle ABC , points D , E , and F lie on BC , AC , and AB respectively such that AD , BE , and CF are concurrent. Suppose AB , AC , and BC have lengths 13, 14, and 15, respectively. If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{CE}{EA} = \frac{5}{8}$, find BD and DC . (Hint: Ceva's)
2. **djmathman** Triangle ABC has $AB = 2007$ and $AC = 2015$. The incircle ω of the triangle is tangent to AC and AB at E and F respectively, and P is the intersection point of EF and BC . Suppose B is the midpoint of \overline{CP} . Compute the length BC . (Hint: Menelaus on $\triangle ABC$ with transversal EF)
3. **Purple Comet 2014 #25** The diagram below shows equilateral $\triangle ABC$ with side length 2. Point D lies on ray \overrightarrow{BC} so that $CD = 4$. Points E and F lie on \overline{AB} and \overline{AC} , respectively, so that E , F , and D are collinear, and the area of $\triangle AEF$ is half of the area of $\triangle ABC$. Find $\frac{AE}{AF}$. (Hint: Menelaus with triangle ABC and transversal EF)



4. In triangle ABC , points D and E lie on side AB dividing the side in a ratio of 1:2:1; in other words, DE is twice as long as AD and EB . Let AM be the median to BC , and let G and H denote the intersection points of this with CD and CE , respectively. What is the ratio $AG : GH : HM$? (Hint: Menelaus on $\triangle AMB$ first with transversal EC and then with transversal DC)
5. **AIME II 2011 #4** In triangle ABC , $AB = \frac{20}{11}AC$. The angle bisector of $\angle A$ intersects BC at point D , and point M is the midpoint of AD . Let P be the point of the intersection of AC and BM . Find the ratio of CP to PA . (Hint: Menelaus on $\triangle ACD$ with transversal PB)
6. **ARML 2012** Given noncollinear points A , B , C , segment AB is trisected by points D and E , and F is the midpoint of segment AC . DF and BF intersect CE at G and H , respectively. If $[EDG] = 18$, compute $[FGH]$. (Hint: Area lemma on CDF and EDF and then do it again for CBF and EBF .)